THE INFLUENCE OF LINEAR ANISOTROPIC SCATTERING OF ONE-SPEED NEUTRONS ON THE CRITICAL SIZE OF A SLAB WITH REFLECTIVE BOUNDARY CONDITIONS

by

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The criticality problem for one-speed neutrons in a slab is investigated using Chebyshev polynomials of first kind in the series expansion of the neutron angular flux in stationary neutron transport equation. The medium is assumed to let the neutrons to scatter anisotropically and to be surrounded by a reflector. The critical thicknesses for the neutrons in a uniform finite slab are computed for selected values of the reflection coefficient and the anisotropy parameter and they are given in the tables. The numerical results obtained from the present method are in good accordance with the results already existed in literature.

Key words: linear anisotropic scattering, Chebyshev polynomial expansion, critical slab, reflective boundary condition

INTRODUCTION

It is important to develop a method to solve the neutron transport equation which describes the behavior and the conservation of the neutrons. These descriptions are very important to operate a nuclear system safely. Spherical harmonics or commonly known as the P_N method is one of the most effective and preferred methods established for the solution of the transport equation. In this method, the neutron angular flux is expanded in terms of the Legendre polynomials. However, it is not the unique and valid one for the solutions of the problems in neutron transport theory. Furthermore, insufficiencies of the P_N method are reported in some problems like Milne problem and anisotropic scattering cases [1, 2]. Therefore, in some earlier studies, other than the Legendre polynomials, the neutron angular flux was expanded in terms of the Chebyshev polynomials of the first kind (T_N method) for the criticality and the extrapolated end point calculations in neutron transport theory [1-3]. In some recent studies, T_N method has been successfully applied to transport equation for the critical thickness of the bare and reflected slabs [4, 5]. In addition, in order to develop new techniques with parameters that are better representing the real system for the solution of the transport equation, the T_N method is revised and applied to criticality problem in transport theory successfully [6-8]. In those studies, numerical results obtained by the revised T_N method for the critical size of the system, were given with the results that existed in literature. Since a good accordance between them is observed, it can be worthwhile to use this revised method for the solution of the transport equation. It is thought that this method can be applied to other problems in science and engineering by the researchers interested in this subject [7, 9].

Since the neutrons migrate anisotropically in real reactor systems, an extended knowledge about the scattering of them through the media should be taken into consideration to attain the solution of the transport equation properly. In other words, anisotropic scattering is an important phenomenon in the solution algorithm of the transport equation. Moreover, reflectors used to reflect the neutrons which are preferably made from fertile or heavy materials, tend to escape from the system back through the core of the reactors. By this way, the neutron population of the system can be conserved to continue the fission chain reaction. Therefore in this study, a modified version of the T_N method is used for the solution of the criticality problem in one-speed and one-dimensional neutron transport theory, using reflective boundary condition together with the Marshak boundary condition [4, 7, 8].

In this method, first, the neutron angular flux is expanded in terms of the Chebyshev polynomials of first kind, as previously used in the studies [6-8]. Then, the flux moment equations are obtained by the proce-

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dure of the method to calculate the eigenvalues. The critical thicknesses of the slab for one-speed neutrons are computed for selected values of the mean number of secondary neutrons per collision c, and the reflection coefficient *R*.

The numerical results for the critical thickness of the slab are given in the tables together with the results obtained by the conventional P_N method, and the ones obtained by Atalay using Case's singular eigenfunctions method [10]. It can easily be seen from the derivations of the equations and the results in the tables that this method has practical executable equations with its rapid convergence. Therefore, the present method can be thought as an alternative method for the solution of the problems which are solved inefficiently by the conventional P_N method in the transport theory.

THEORY AND EQUATIONS

The stationary transport equation for one-speed neutrons travelling in direction Ω' before and Ω after a scattering collision and no external neutron sources can be written as,

$$\Omega \cdot \nabla\psi(r,\Omega) + \sigma_T\psi(r,\Omega) = c\sigma_T \int \psi(r,\Omega')f(\Omega'\cdot\Omega)\mathrm{d}\Omega' \tag{1}$$

where $\psi(r,\Omega)$ is the angular flux of the neutrons at position *r*, $f(\Omega'\Omega)$ – the scattering function which describes the interaction of the neutrons with fuel and other material atoms inside the system, and σ_T – the total macroscopic cross-section [11].

For 1-D case, the transport equation with linear anisotropic scattering for one-speed neutrons can be written

$$\mu\frac{\partial\psi(x,\mu)}{\partial x} + \sigma_T\psi(x,\mu) = \frac{c\sigma_T}{2}\int_{-1}^{1}\psi(x,\mu')(1+3b_1\mu\mu')\mathrm{d}\mu' \tag{2}$$

with free space boundary and symmetry conditions

$$\psi(a,\mu) = 0 \tag{3a}$$

$$\psi(x,\mu) = \psi(-x,-\mu), \quad \mu > 0 \tag{3b}$$

Here, the slab is thought to be finite, homogeneous with a thickness of $2a$ extending from $x = -a$ to $x = a$ in units of mean free path. Besides, it is surrounded by identical reflectors from both sides. $\psi(x,\mu)$ is the angular flux of the neutrons at position *x* travelling in direction μ – cosine of the angle between the neutron velocity vector and the positive x-axis, as can be expected. b_1 is the average cosine of the scattering angle, $|b_1| \le 1/3$ [12].

By following the studies previously carried out by the authors [6-8], the neutron angular flux is expanded as

$$\psi(x,\mu) = \frac{1}{\pi}\sum_{n=0}^{N}(2-\delta_{n,0})\Phi_n(x)T_n(\mu),$$
$$-a \le x \le a, \quad -1 \le \mu \le 1 \tag{4}$$

where $T_n(\mu)$ is the n^{th} order Chebyshev polynomials of first kind representing the angular part of the neutron angular flux and $\Phi_n(x)$ – the moments of angular flux. When eq. (4) is substituted into eq. (1) using the orthogonality and recurrence relations of the Chebyshev polynomials of first kind, respectively [13]

$$\int_{-1}^{1}\frac{(2-\delta_{n,0})T_n(\mu)T_m(\mu)}{\pi\sqrt{1-\mu^2}}\mathrm{d}\mu = \delta_{m,n} \tag{5}$$

$$T_{n+1}(\mu) - 2\mu T_n(\mu) + T_{n-1}(\mu) = 0 \tag{6}$$

One can obtain the T_N moments of the angular flux

$$\frac{\mathrm{d}\Phi_1(x)}{\mathrm{d}x} + \sigma_T(1-c)\Phi_0(x) = 2c\sigma_T\sum_{n=1}^{N}\frac{\Phi_{2n}(x)}{4n^2-1}, \quad n = 0 \tag{7a}$$

$$\frac{\mathrm{d}\Phi_2(x)}{\mathrm{d}x} + \frac{\mathrm{d}\Phi_0(x)}{\mathrm{d}x} + 2\sigma_T\Phi_1(x) = 6b_1c\sigma_T\left[\frac{1}{3}\Phi_1(x) + \sum_{n=1}^{N}\frac{\Phi_{2n-1}(x)}{(2n-1)^2-4}\right], \quad n = 1 \tag{7b}$$

and in general,

$$\frac{\mathrm{d}\Phi_{n+1}(x)}{\mathrm{d}x} + \frac{\mathrm{d}\Phi_{n-1}(x)}{\mathrm{d}x} + 2\sigma_T\Phi_n(x) = 0, \quad n \ge 2 \tag{7c}$$

The solutions for eq. (7) are established in the form [11]

$$\Phi_n(x) = G_n(\nu)\exp\left(-\frac{\sigma_T x}{\nu}\right) \tag{8}$$

In order to obtain a system of equations for the eigenvalues, one should replace eq. (8) in eq. (7) to get

$$G_1(\nu) - \nu(1-c)G_0(\nu) = -2\nu c\sum_{n=1}^{N}\frac{G_{2n}(\nu)}{4n^2-1}, \quad n = 0 \tag{9a}$$

$$G_2(\nu) + G_0(\nu) - 2\nu G_1(\nu) = -6b_1\nu c\left[\frac{1}{3}G_1(\nu) + \sum_{n=1}^{N}\frac{G_{2n-1}(\nu)}{(2n-1)^2-4}\right], \quad n = 1 \tag{9b}$$

$$G_{n+1}(\nu) + G_{n-1}(\nu) - 2\nu G_n(\nu) = 0, \quad n \ge 2 \tag{9c}$$

where $G_{-1}(\nu) = 0$ and $G_0(\nu) = 1$ are taken so and by following the same procedure as in P_N approximation [14], the discrete and continuum eigenvalues can be obtained by setting $G_{N+1}(\nu) = 0$ in eqs. (9) for the N^{th} order approximation of the present method. In particular, for low order approximations, eqs. (9) can be carried out manually and analytic expressions for the eigenvalues can be found and they can be calculated for various values of c and b_1. However, for higher order approximations, since the number equations increases, it becomes difficult to get eigenvalues from eqs. (9). Therefore, a matrix notation can be preferred to calculate the eigenvalues for higher order approximations and so eqs. (9) can be written in a matrix form

$$[\mathbf{M}(\nu)]\mathbf{G}(\nu) = \mathbf{0} \tag{10}$$

where $\mathbf{M}(\nu)$ is the $(N+1) \times (N+1)$ coefficient matrix and $\mathbf{G}(\nu)$ is a column matrix, $\mathbf{G}(\nu) = [G_0(\nu), G_1(\nu), \ldots G_N(\nu)]^T$. The matrix equation given in eq. (10) represents a system of linear homogeneous equations. Since setting $\mathbf{G}(\nu) = 0$ gives an undesired trivial solution, for a non-trivial solution for the discrete eigenvalues, the determinant of the coefficient matrix should be equal to zero, *i. e.*, det $[\mathbf{M}(\nu)] = 0$.

The discrete eigenvalues are computed using selected orders of T_N approximation from eq. (10) for various values of c and b_1. Since an eigenfunction corresponds to each ν_k eigenvalues, a linear combination of the eigenfunctions should also be a general solution for the flux moments for odd numbers of N

$$\Phi_n(x) = \sum_{k=1}^{\frac{N+1}{2}} \lambda_k G_n(\nu_k) \left[\exp\left(-\frac{\sigma_T x}{\nu_k}\right) + (-1)\exp\left(\frac{\sigma_T x}{\nu_k}\right)\right], \quad n = 1,3,\ldots,N \tag{11}$$

The linear combination constants λ_k's can be determined from the physical boundary of the system, and the parity relation of $G_n(-\nu) = (-1)^n G_n(\nu)$ is used. However, in this problem there is no need to calculate the λ_k's, since they are disappeared in the criticality condition. Therefore, when the flux moments given in eq. (11) are replaced in eq. (4), one can obtain the general solution for the transport equation (eq. (1))

$$\psi(x,\mu) = \frac{2}{\pi}\sum_{k=1}^{\frac{N+1}{2}} \left\{ \lambda_k G_0(\nu_k) T_0(\mu) \cosh\left(\frac{\sigma_T x}{\nu_k}\right) + \sum_{n=1}^{N} \lambda_k G_n(\nu_k) \left[\exp\left(-\frac{\sigma_T x}{\nu_k}\right) + (-1)\exp\left(\frac{\sigma_T x}{\nu_k}\right)\right] T_n(\mu) \right\} \tag{12}$$

BOUNDARY AND CRITICALITY CONDITIONS

In one of the studies about the inabilities of the P_N method in some circumstances, it is stated that the P_N approximation in slab geometries is a rather poor representation of the angular flux near material boundaries [15]. Experience indicates that the Marshak boundary conditions are somewhat more accurate than the Mark conditions, at least for small N [14]. Therefore in this study, the Marshak boundary condition which is based on the condition of zero incoming current at the vacuum boundary is preferred to use for the calculation of the critical size of the slab surrounded by identical reflectors from both sides. It is therefore for T_N approximation [14]

$$\int_0^1 [\psi(a,-\mu) - R\psi(a,\mu)] T_k(-\mu)\,d\mu = 0, \quad k = 1,3,5,\ldots,N \tag{13}$$

The criticality equation can be obtained by using eq. (12) in eq. (13) with the parity relation of the Chebyshev polynomials of first kind; $T_k(\mu) = (-1)^k T_k(-\mu)$

$$\sum_{k=1}^{\frac{N+1}{2}} (-1)^k \lambda_k \left\{ G_0(\nu_k)[1-R]\cosh\left(\frac{\sigma_T a}{\nu_k}\right) I_k + \sum_{n=1}^{N} G_n(\nu_k) \left[[(-1)^n - R]\left[(1+(-1)^n)\cosh\left(\frac{\sigma_T a}{\nu_k}\right) - (1-(-1))\sinh\left(\frac{\sigma_T a}{\nu_k}\right)\right] I_{n,k} \right] \right\} = 0 \tag{14}$$

where the integrands I_k and $I_{n,k}$ are defined as,

$$\int_0^1 T_k(-\mu)\,d\mu = (-1)^k I_k \tag{15}$$

$$I_k = \int_0^1 T_k(\mu)\,d\mu = \begin{cases} 1, & k = 0, \\ 1/2, & k = 1, \\ \dfrac{k\sin(k\pi/2) - 1}{k^2 - 1}, & k \geq 2 \end{cases} \tag{16}$$

$$I_{n,k} = \int_0^1 T_n(\mu) T_k(\mu)\,d\mu = \begin{cases} 1, & n = k = 0, \\ n\cos(n\pi)/(4n^2 - 1), & n \; k \; 0, \\ \dfrac{\varphi(k,n)(n^2 + k^2 - 1)}{(k^2 - n^2)^2 - 2(k^2 + n^2) + 1}, & n \neq k, \end{cases} \tag{17}$$

$$\int_0^1 T_k(-\mu) T_n(-\mu)\,d\mu = (-1)^{n+k} I_{n,k} \tag{18}$$

and

$\varphi(k,n) = [k\sin(k\ /2)\cos(n\ \ n\sin(n\ \ \cos(k$
$n^2\ k -$

A similar matrix representation can be carried out for the criticality equation as for the calculation of the eigenvalues given in eqs. (9). Thus, eq. (14) can also be written in the matrix form as,

$$[\mathbf{M}_n^k(a)]\Lambda_k = \mathbf{0} \quad n = 1,2,\ldots,(N+1)/2, \quad k = 1,2,\ldots,(N+1)/2 \tag{19}$$

where $\mathbf{M}_n^k(a)$ is the coefficient matrix including the parameters of the critical half-thickness a, eigenvalues $_k$, and collision and scattering parameters with $(N+1)/2 \times (N+1)/2$ elements, Λ_k is the column vector with elements of linear combination constants $[\lambda\ \ \lambda$

λ $]^T$ and **0** is a null vector. By following the same procedure as in eq. (10), a non-trivial solution of eq. (19) can be found by equating the determinant of the coefficient matrix to zero, *i. e.* $\det[\mathbf{M}_n^k(a)] \; 0$. For instance, for T_1 approximation, an analytic expression for the critical half-thickness of the reflected slab can easily be obtained by setting $N = 1$ in eq. (14) or eq. (19)

$$a \quad \frac{1}{\sigma_T\sqrt{2(1 \; c)(1 \; b_1 c)}}\tanh^{\;1}\left(\frac{3}{2}\sqrt{\frac{1 \; b_1 c}{2(1 \; c)}}\frac{1 \; R}{1 \; R}\right) \tag{20}$$

NUMERICAL RESULTS

The reflected critical slab problem for one-speed neutrons in an uniform finite slab of thickness $2a$ is studied in the case of linear anisotropic scattering. Modified T_N method, previously carried out by the authors [6-8], is used for the solution of the problem using Marshak boundary conditions for various values of c, b_1, and R. During the calculations, the total macroscopic cross-section is assumed to be its normalized value, *i. e.* $\sigma_T = 1$ cm^{-1} and all the numerical results for both eigenvalues and critical thicknesses are performed using Maple software.

In the method, first the neutron angular flux is expanded in a series the Chebyshev polynomials of first kind as in previous works [6-8] and then T_N moments of the flux are obtained. After a reasonable solution is presented for the scalar neutron flux in eq. (8), the discrete eigenvalues are computed by setting $G_{N+1}(\nu) = 0$ in eqs. (9) or eq. (10) for various values of c and b_1. Finally, the critical thicknesses of the slab are computed by using the criticality equation given in eq. (14) or the matrix equation in eq. (19). The calculations are maintained up to the order of T_9 approximation which is reported as to be efficient in the case of using Marshak boundary conditions [14].

The critical thicknesses for one-speed neutrons are calculated using the present T_N method for typical values of the cross-section parameter c, from 1.01 to 2.00, anisotropy parameter b_1, from –0.3 to 0.3 and the reflection coefficient R, from 0.00 to 0.99. Moreover, the results obtained from the present method are tabulated in the tables together with the results obtained from the traditional P_N method and the ones from literatures for comparison. While the exact results presented in literature are quoted from Lee-Dias and Aranson [15, 16], the other results are from the study of Atalay [10].

The critical thicknesses for one-speed neutrons in a bare slab ($R = 0.00$) calculated by the present T_N method are given in tab. 1 for various values of the cross-section and anisotropy parameters. In this table, the results obtained by the present method, as well as the results obtained for the isotropic scattering quoted from refs. [15, 16], by Case's singular eigenfunction method quoted from Atalay [10] and by the traditional P_N method, are given. Therefore, a comprehensive comparison can be done between the results obtained from the present T_N method and the results that already exist in literature. In addition, from the first table through the last one, one can easily realize the effectiveness of the method used in this study by observing the good accordance between the results. Meanwhile, the critical thicknesses for one-speed neutrons in a reflected slab are calculated for the same cross-section and anisotropy parameters with the reflection coefficients R changing from 0.25 to 0.99. Those results are also given in tabs. 2, 3, and 4.

It is stated, in *Theory and equations* part of this study, that the transport equation is established under consideration of a system without a source and thus, the

Table 1. The critical slab thicknesses as calculated by T_9 approximation, compared with P_9 and literature ($R = 0.00$)

	b_1	c				
		1.01	1.10	1.20	1.40	2.00
T_9	–0.3	14.74922	3.81288	2.35148	1.36304	0.59485
	0.0	16.66256	4.23008	2.58228	1.47717	0.63120
	0.1	17.49315	4.40628	2.67813	1.52347	0.64532
	0.2	18.46632	4.60899	2.78720	1.57533	0.66071
	0.3	19.62842	4.84592	2.91302	1.63408	0.67758
P_9	–0.3	14.74778	3.81156	2.35024	1.36166	0.59212
	0.0	16.66068	4.22842	2.58076	1.47566	0.62836
	0.1	17.49194	4.40446	2.67648	1.52186	0.64242
	0.2	18.46487	4.60697	2.78538	1.57364	0.65775
	0.3	19.62568	4.84364	2.91098	1.63226	0.67458
Atalay [10]	0.0	16.65904	4.22674	2.57968	1.47688	0.63258
	0.1	17.48930	4.40260	2.67530	1.52390	0.65236
	0.2	18.46196	4.60486	2.78406	1.57676	0.67816
	0.3	19.62374	4.84124	2.90948	1.63694	0.71692
Exact [14, 15]	0.0	16.65902	4.22662	2.57876	1.47320	0.62206

Table 2. The critical slab thicknesses as calculated by T_9 approximation, compared with P_9 and literature ($R = 0.25$)

	b_1	c				
		1.01	1.10	1.20	1.40	2.00
T_9	–0.3	14.06923	3.26999	1.90829	1.04070	0.42121
	0.0	15.78167	3.54555	2.03603	1.09221	0.43402
	0.1	16.51606	3.65646	2.08574	1.11154	0.43864
	0.2	17.36948	3.78005	2.13996	1.13218	0.44344
	0.3	18.37830	3.91906	2.26063	1.15427	0.44846
P_9	–0.3	14.06644	3.26784	1.90648	1.03890	0.41866
	0.0	15.77810	3.54294	2.03392	1.09030	0.43140
	0.1	16.51268	3.65364	2.08352	1.10958	0.43596
	0.2	17.36563	3.77698	2.13759	1.13016	0.44073
	0.3	18.37350	3.91570	2.19692	1.15220	0.44570
Atalay [10]	0.0	15.74156	3.51332	2.01042	1.07260	0.42806
	0.1	16.47158	3.62050	2.05682	1.09348	0.43550
	0.2	17.31947	3.73955	2.10703	1.11182	0.44589
	0.3	18.32168	3.87306	2.16166	1.13110	0.46330

Table 3. The critical slab thicknesses as calculated by T_9 approximation, compared with P_9 and literature (R = 0.75)

	b_1	c				
		1.01	1.10	1.20	1.40	2.00
T_9	–0.3	9.55886	1.29242	0.64835	0.31410	0.11432
	0.0	10.17593	1.31066	0.65351	0.31552	0.11458
	0.1	10.41385	1.31694	0.65527	0.31600	0.11467
	0.2	10.67203	1.32334	0.65704	0.31648	0.11476
	0.3	10.95280	1.32985	0.67476	0.31696	0.11485
P_9	–0.3	5.54960	1.29012	0.64690	0.31306	0.11344
	0.0	10.16518	1.30828	0.65204	0.31448	0.11370
	0.1	10.40242	1.31454	0.65378	0.31494	0.11378
	0.2	10.65975	1.32091	0.65554	0.31542	0.11387
	0.3	10.94038	1.32740	0.65732	0.31590	0.11396
Atalay [10]	0.0	9.89338	1.24756	0.62088	0.30064	0.05535
	0.1	10.11262	1.25104	0.62064	0.30000	0.11132
	0.2	10.34944	1.25442	0.62022	0.29918	0.11260
	0.3	10.60644	1.25774	0.61960	0.29818	0.11550

Table 4. The critical slab thicknesses as calculated by T_9 approximation, compared with P_9 and literature (R = 0.99)

	b_1	c				
		1.01	1.10	1.20	1.40	2.00
T_9	–0.3	0.50137	0.04879	0.02371	0.01129	0.00406
	0.0	0.50145	0.04879	0.02371	0.01129	0.00406
	0.1	0.50148	0.04879	0.02371	0.01129	0.00406
	0.2	0.50152	0.04879	0.02371	0.01129	0.00406
	0.3	0.50151	0.04879	0.02371	0.01129	0.00406
P_9	–0.3	0.50038	0.04868	0.02365	0.01125	0.00402
	0.0	0.50046	0.04868	0.02365	0.01125	0.00402
	0.1	0.50050	0.04868	0.02365	0.01125	0.00402
	0.2	0.50054	0.04868	0.02365	0.01125	0.00402
	0.3	0.50056	0.04868	0.02365	0.01125	0.00402
Atalay [10]	0.0	0.46986	0.04582	0.02234	0.01070	0.00392
	0.1	0.46978	0.04574	0.02228	0.01066	0.00394
	0.2	0.46971	0.04565	0.02220	0.01062	0.00398
	0.3	0.46964	0.04556	0.02212	0.01056	0.00408

number of neutrons propagated in the system is assumed to be conserved. Since then, it is seen from the numerical results given in the tables, the critical thickness of the slab decreases with increasing values of the reflection coefficient and it approaches to zero when the reflection coefficient goes to unity, as expected. This manner can be explained physically that the distribution of the neutrons in the system is said to be completely dense in the normal plane, x = 0. Another point observed in the tables is that the critical thickness of the slab decreases when the cross-section parameter c increases. It can also be said that the critical thickness approaches to zero with increasing values of the cross-section and anisotropy parameters together with the increasing values of the reflection coefficient, as expected. This zero value of the critical thickness is not achieved in this study but, it can be asserted that it is possible if the order of the approximation is increased to $N > 9$. However, the critical thickness of the slab increases with the increasing values of the anisotropy parameter b_1 ranging from –0.3 to +0.3 for all values of the cross-section parameter c. In other words, the critical size of a system with reflectors, where the neutrons are thought to scatter anisotropically, represents the monotonic behavior.

Another important point that will come from the tables is that the numerical results for the critical thickness, obtained from the present method, are seen to be approximately the same with the ones obtained from P_N method and the exact ones. This confirmation is expected since both the Legendre and Chebyshev polynomials take place in the same family, *i. e.* Jacobi polynomials. Therefore, as claimed in the Introduction, the T_N method is an alternative to traditional P_N method, with their very similar results given in the tables.

CONCLUSION

In this study, a modified version of the T_N method, previously applied by the authors [6-8] in the solution algorithm of the transport equation, is used for the reflected critical slab problem with linear anisotropic scattering. As in their studies, the numerical results obtained by the present method are in good accordance with the ones already presented in literature. This is expected since the Chebyshev and Legendre polynomials are the members of the same family, *i. e.* Jacobi polynomials. Since the results tabulated in comparison with the literature values indicate the applicability and the effectiveness of the present method, one can easily conclude that an alternative technique is derived and it can be a source of inspiration for the researchers to solve or complete other problems in science and engineering.

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Хакан ОЗТУРК

УТИЦАЈ ЛИНЕАРНО АНИЗОТРОПНОГ РАСЕЈАЊА МОНОЕНЕРГЕТСКИХ НЕУТРОНА НА КРИТИЧНОСТ ПЛОЧЕ СА РЕФЛЕКТИВНИМ ГРАНИЧНИМ УСЛОВИМА

Коришћењем развоја неутронског угаоног флукса у Чебишевљеве полиноме прве врсте, истражена је критичност плоче помоћу временски независне моноенергетске транспортне једначине. Претпостављено је да је медијум окружен рефлектором и да се неутрони расејавају анизотропно. Критичне дебљине униформне ограничене плоче, израчунате за одабране вредности коефицијента рефлексије и параметра анизотропије, приказане су табеларно. Нумерички резултати добијени приказаном методом добро се слажу са подацима наведеним у литератури.

Кључне речи: линеарно анизотропно расејање, Чебишевљев полином, критична плоча, рефлексивни гранични услов